

PHY 489 Week 13:

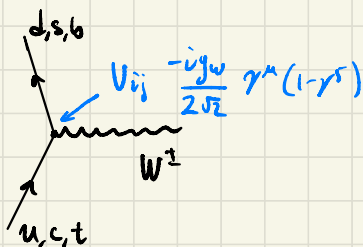
Neutrino Oscillations; The Higgs Mechanism

Neutrino Oscillations and Masses

Recall CKM matrix for quark flavor changing:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

at interaction vertices



Similarly, same situation arises with neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

← Mass eigenstates

- Also recall time-dependence which arises from the Schrödinger equation

$$|\psi(t)\rangle = |\psi(0)\rangle e^{iEt/\hbar}$$

A neutrino in a mass eigenstate will stay in that eigenstate, i.e. a flavour state is a linear combination of mass eigenstates:

$$|\nu_\alpha\rangle = u_{\alpha 1} |\nu_1\rangle + u_{\alpha 2} |\nu_2\rangle + u_{\alpha 3} |\nu_3\rangle$$

for $\alpha = e, \mu, \tau$.

For a neutrino in its rest frame evolving at the proper time τ , the state then evolves as

$$|\nu_\alpha\rangle = \sum_k u_{\alpha k} e^{-im_k \tau} |\nu_k\rangle$$

Proper time τ , mass m is elapsed energy in rest frame of particle

$$m_k \tau = p \cdot x = E_k t - \vec{p}_k \cdot \vec{x}$$

so that in other reference frames we find that

$$|\nu_\alpha\rangle = \sum_K u_{\alpha K} e^{-i(E_K t - \vec{p}_K \cdot \vec{x})} |\nu_i\rangle$$

- Suppose that the flavour state is composed of mass states all with energy E and $E \gg m_K$,

$$p_K = \sqrt{E^2 - m_K^2} \approx E \left(1 - \frac{m_K^2}{2E^2}\right)$$

so the state evolves as

$$\begin{aligned}
 |\nu_\alpha\rangle &= \sum_K u_{\alpha K} e^{-i(E_K(t-L) + \frac{m_K^2}{2E}L)} |\nu_K\rangle \\
 &= \sum_K u_{\alpha K} e^{-i\frac{m_K^2}{2E}L} |\nu_K\rangle
 \end{aligned}$$

↑ overall phase
↑ L : spatial parameter, dimensions length

The amplitude of a measurement between two states

$|\nu_\alpha\rangle \rightarrow |\nu_\beta\rangle$ is then

$$\begin{aligned}
 \langle \nu_\beta | \nu_\alpha \rangle &= \sum_K u_{\alpha K} e^{-i\frac{m_K^2}{2E}L} \langle \nu_\beta | \nu_K \rangle \\
 &= \sum_K u_{\alpha K} u_{\beta K}^* e^{-i\frac{m_K^2}{2E}L}
 \end{aligned}$$

↑ $u_{\beta K}^*$

And squaring gives the probability

$$P_{\alpha \rightarrow \beta}(L) = |\langle \nu_\beta | \nu_\alpha \rangle|^2$$

$$\Delta m_{\alpha\beta}^2 = m_\alpha^2 - m_\beta^2$$

$$= \delta_{\alpha\beta} - 4 \operatorname{Re} \left\{ \sum_{k>l} U_{\alpha k} U_{\beta k}^* U_{\alpha l} U_{\beta l}^* \right\} \left[\sin^2 \frac{\Delta m_{\alpha\beta}^2 L}{4E} \right]$$

Wavelength

$$+ 2 \operatorname{Im} \left\{ \sum_{k>l} U_{\alpha k} U_{\beta l}^* U_{\alpha l} U_{\beta k}^* \right\} \left[\sin^2 \frac{\Delta m_{\alpha\beta}^2 L}{4E} \right]$$

Amplitude

prevents double-counting

Oscillations:
probability is
sinusoidal in $\frac{L}{E}$

Two-Flavor model:

For e, μ leptons, we can write

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Where $\cos \theta, \sin \theta$ are just coefficients that enforce normalization. Then

$$P_{\mu \rightarrow e} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\Delta m_{21}^2 = m_e^2 - m_\mu^2$$

$$P_{\mu \rightarrow \mu} = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

← conserve probability

$$P_{\mu \rightarrow e} + P_{\mu \rightarrow \mu} = 1$$

• Neutrino Sources

- Nuclear fission/fusion

- Solar energy (3% of sun radiation is neutrinos)

- Meson/muon decays, atmospheric neutrinos

Energy:

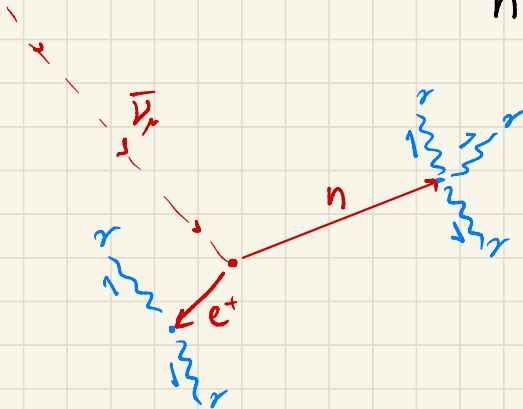
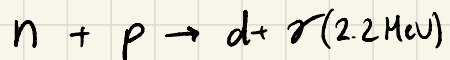
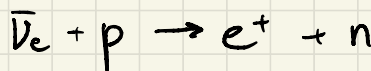
$\sim (MeV)$

$\sim (GeV)$

• Detection

- Large volume to gather neutrino interactions

- Antineutrinos: "inverse β^- -decay"



Detectors usually consist of H_2O or D_2O (heavy water) in below-ground acrylic/steel/PVC casings, surrounded by photomultiplier tubes to detect γ emissions.

Review of Classical Field Theory

- Lagrangian mechanics: $L = T - V \quad (q, \dot{q})$

Principle of least action $\delta S = \delta \int_t^{+t} L(q, \dot{q}, t) dt = 0$

implies "Euler-Lagrange" equations $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) = \frac{\partial L}{\partial q}$

- Classical Field Theory Formulation: $q(t) \rightarrow \phi(x)$

so $L \rightarrow \mathcal{L}(\phi(x), \partial_\mu \phi(x))$

← Lagrange density: $\mathcal{L} = \int d^3x \mathcal{L}$

Hence, by $\delta S = 0$ we obtain the EL equations for fields

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i}$$

Common Lagrangians and their EOM:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \quad \xleftrightarrow{\text{Klein-Gordon}} \quad \partial_\mu \partial^\mu \phi + m^2 \phi = 0. \quad \text{Spin } 0$$

$$\mathcal{L} = i \bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi \quad \xleftrightarrow{\text{Dirac}} \quad i \not{\partial} \psi - m \psi = 0 \quad \text{Spin } \frac{1}{2}$$

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{m^2}{8\pi} A^\mu A_\mu \quad \xleftrightarrow{\text{Proca}} \quad \partial_\mu F^{\mu\nu} + m^2 A^\nu = 0 \quad \text{Spin } 1$$

Local Gauge Invariance of Fields

- Dirac equation invariant under global phase transformation of the spinor $\psi \rightarrow e^{i\theta} \psi$ and $\theta \in \mathbb{R}$.

If the phase factor θ is now different at each spacetime point $\theta(x^\mu)$ we obtain the local phase transformation

$$\psi \rightarrow e^{i\theta(x^\mu)} \psi \quad (\text{spinor fields})$$

$$\rightarrow \partial_\mu \psi = e^{i\theta(x^\mu)} (\partial_\mu \psi) + e^{i\theta(x^\mu)} \partial_\mu \psi$$

from which

$$\mathcal{L} \rightarrow \mathcal{L} - \partial_\mu \theta(x^\mu) \bar{\psi} \gamma^\mu \psi.$$

Instead let:

$$\lambda(x^\mu) = -\frac{1}{q} \theta(x^\mu), \quad q: \text{particle charge then}$$

$$\text{sends } \psi \rightarrow e^{-iq\lambda} \psi \text{ and } \mathcal{L} \rightarrow \mathcal{L} + q(\bar{\psi} \gamma^\mu \psi) \partial_\mu \lambda$$

fields unique up to a derivative of some function $\lambda(x^\mu) \dots$

- The Gauge transformation of the fields

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda \quad (\text{vector field local invariance by } \lambda(x^\mu))$$

Now leaves the Dirac Lagrangian density locally invariant, but

for fields with bilinears of the fields (ie, the Proca "mass terms"

$\frac{m^2}{2} A^\mu A_\mu$) is still not invariant, $m=0$ is required for

invariance. $\rightarrow$ Forced to introduce massless vector field...

* Classically, in EM, A^μ is electromagnetic potential 4-vector.

The total Gauge symmetry requires massless vector field,

$$\mathcal{L} = [i \bar{\psi} \not{\partial} \psi - m \bar{\psi} \psi] - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - (g \bar{\psi} \gamma^\mu \psi) A_\mu$$

g : some perturbation parameter $\sim$ coupling between spinor and massless vector field

* Determine common Gauge transformation for both ψ , A^μ fct'n of λ .

The local Gauge invariance of the spinor field (Dirac eqn) ψ is then

$$\partial_\mu \psi \rightarrow e^{-i g \lambda} [\partial_\mu - i g \partial_\mu \lambda] \psi$$

ONLY FOR
MASSLESS VECTOR
FIELDS

and we define the "Covariant Derivative" $\mathcal{D}_\mu = \partial_\mu + i g A_\mu$

which takes into account the local transformation of A_μ

and

$$\mathcal{D}_\mu \psi \rightarrow e^{-i g \lambda} \mathcal{D}_\mu \psi$$

"Minimal Coupling
Rule"

so the invariance of $\mathcal{L}$ is restored.

• In general, we can think of local Gauge invariance as a $U(1)$ symmetry $\psi \rightarrow U \psi$ with $U^\dagger U = 1$

$\uparrow$ some $e^{i\theta}$

The Mass Term

- Gauge theories are renormalizable ; the mass terms in the Klein-Gordon / Proca equation are not locally invariant.

Photons / Gluons OK (Massless)

$W^\pm / Z^0$ NOT OK (Massive)

Mass terms in electroweak theory via chiral fermion coupling $m \bar{\psi} \psi$ NOT OK

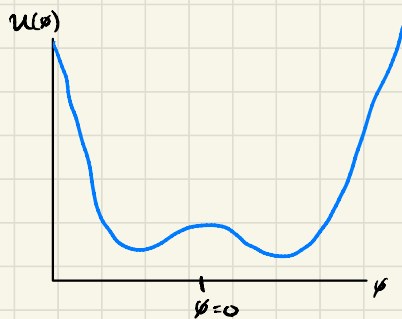
(Symmetry $U(2) \rightarrow U(1)$ violation)

How to reconcile?

- Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\mu^2}{2} \phi^2 - \frac{\lambda}{4!} \phi^4$$

when $\phi=0$ is not a stable configuration of interactions.



$\therefore$ Lowest energy state occurs when $\phi \neq 0$

Feynman calculus - perturbation theory of fields acting on Vacuum state $|0\rangle$
to work

We can see that the minimum occurs when $\phi = \pm \frac{\mu}{\lambda}$,
 from which we can define the new field $z = \phi \pm \frac{\mu}{\lambda}$
 which has the appropriate ground-state values, and the Lagrangian
 becomes



$$\mathcal{L} = \frac{1}{2} (\partial_\mu z) (\partial^\mu z) - \mu^2 z^2 \pm \mu \lambda z^3 - \frac{1}{4} \lambda^4 z^4 + \frac{1}{4} \left(\frac{\mu^2}{\lambda} \right)^2$$

$\uparrow$ mass term
 $m = \sqrt{2} \mu$
 $\uparrow$ correct sign

and this is the standard procedure for determining mass terms.

Spontaneous Symmetry Breaking

- Last example: discrete symmetry breaking.

Here: continuous symmetry breaking.

Consider the two-scalar field Lagrangian

invariant under rotations in ϕ_1 - ϕ_2 field space.

$$\phi_1^2 + \phi_2^2 = \frac{\mu^2}{\lambda^2}$$

Minimum condition
on ϕ_1, ϕ_2 .

Expand about ground/vacuum state with fields

$$\eta = \phi_1 - \frac{\mu}{\lambda}, \quad \xi = \phi_2$$

Then

$$\mathcal{L} = \left[\frac{1}{2} (\partial_\mu \eta) (\partial^\mu \eta) - \overset{m_\eta = \sqrt{2}\mu}{\mu^2 \eta^2} \right] + \left[\frac{1}{2} (\partial_\mu \xi) (\partial^\mu \xi) \right] - \left[\mu \lambda (\eta^3 + \eta \xi^2) + \frac{\lambda^2}{4} (\eta^4 + \xi^4 + 2\eta^2 \xi^2) \right] + \frac{\mu^4}{4\lambda^2}$$

Additional couplings:

Shift in $\mathcal{L}$ negligible

$m_\xi = 0$ (massless)

Higgs Mechanism

- Consider the same Lagrangian density as last example,

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi_1 \partial_\mu \phi_1 + \frac{1}{2} \partial^\mu \phi_2 \partial_\mu \phi_2 + \frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2) - \frac{1}{4} \lambda^2 (\phi_1^2 + \phi_2^2)^2$$

but parametrize the fields into a complex field space

$$\varphi \equiv \phi_1 + i \phi_2$$

so that $\varphi^* \varphi = \phi_1^2 + \phi_2^2$.

Then

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi^*) (\partial^\mu \varphi) + \frac{1}{2} \mu^2 (\varphi^* \varphi) - \frac{1}{4} \lambda^2 (\varphi^* \varphi)^2$$

Observe now that the ϕ_1, ϕ_2 rotational $SO(2)$ symmetry was now broken and cast into a $U(1)$ symmetry under phase transformations

$$\varphi \rightarrow e^{i\theta(x)} \varphi$$

but w/ scalar fields instead of spinors now.

As before, introduce the massless gauge field coupling A^μ via the covariant derivative $\mathcal{D}_\mu = \partial_\mu + i q A_\mu$:

$$\mathcal{L} = \frac{1}{2} \left[\mathcal{D}_\mu^* \varphi^* \mathcal{D}^\mu \varphi \right] + \frac{1}{2} \mu^2 \varphi^* \varphi - \frac{1}{4} \lambda^2 (\varphi^* \varphi)^2 - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

Let $u = \phi_1 - \frac{\mu}{\lambda}$, $x = \phi_2$ to break continuous symmetry...

$$\begin{aligned} \mathcal{L} = & \left[\frac{1}{2} (\partial_\mu u) (\partial^\mu u) - \mu^2 u^2 \right] + \left[\frac{1}{2} (\partial_\mu x) (\partial^\mu x) \right] + \left[-\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left(q \frac{\mu}{\lambda} \right)^2 A_\mu A^\mu \right] \\ & + \left[q \left(u \partial_\mu x - x \partial_\mu u \right) A^\mu + \frac{\mu}{\lambda} q^2 u (A_\mu A^\mu) + \frac{1}{2} \lambda^2 (x^2 + u^2) (A_\mu A^\mu) \right. \\ & \left. - \lambda \mu (u^3 + u x^2) - \frac{\lambda^2}{4} (u^4 + 2 u^2 x^2 + x^4) \right] \\ & + \left(\frac{\mu}{\lambda} q \right) (\partial_\mu x) A^\mu + \left(\frac{\mu^2}{2\lambda} \right)^2 \text{shift} \end{aligned}$$

Nambu-Goldstone

Kinetic

Couplings

The couplings in the Lagrange density are "Gauge couplings"

$$g^2 A_\mu A^\mu \psi^2$$

$$\lambda \frac{\mu}{\lambda} \psi^3$$

$$g^2 \frac{\mu}{\lambda} A_\mu A^\mu \psi$$

$$\frac{1}{4} \psi^4$$

Electroweak Symmetry Breaking

- We consider a weak isodoublet of Higgs fields

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \rightarrow \begin{matrix} Q=1 \\ Q=0 \end{matrix}$$

with hypercharge $Y=1 = 2(Q - I_3)$ and Lagrangian

$$\mathcal{L} = (\partial_\mu \varphi)^\dagger (\partial^\mu \varphi) - \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

$\hookrightarrow \varphi$ has a "ring" of degenerate vacuum states at

$$\varphi^\dagger \varphi = -\frac{\mu^2}{2\lambda} \quad ; \quad \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Introduce Gauge symmetry under $SU(2)_L \times U(1)_Y$

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu + i g_w \vec{\tau} \cdot \vec{W}_\mu + i g' \frac{Y}{2} B_\mu$$

↑
Isospin Pauli
Vector
Matrices
↑ Hypercharge Gauge Boson

$$= \frac{1}{2\sqrt{2}} \begin{pmatrix} i g_w (W_\mu^1 - i W_\mu^2) \\ 2\partial_\mu - i g_w W_\mu^3 + i g' B_\mu \end{pmatrix} (v+h)$$

• Only considering the kinetic term in the Lagrangian:

$$\mathcal{L}_{kin} = (\partial_\mu \varphi)^\dagger (\partial_\mu \varphi) \rightarrow (\mathcal{D}_\mu \varphi)^\dagger (\mathcal{D}_\mu \varphi)$$

$$\text{with } \mathcal{D}_\mu \varphi = \frac{1}{2\sqrt{2}} \begin{pmatrix} i g_w (W_\mu^1 - i W_\mu^2) \\ 2\partial_\mu - i g_w W_\mu^3 + i g' B_\mu \end{pmatrix} (v+h)$$

$$2\partial_\mu \rightarrow \frac{1}{2} (\partial_\mu h) (\partial^\mu h)$$

$$(2) \quad i g_w (W_\mu^1 - i W_\mu^2) \rightarrow \frac{g_w^2}{8} (W_\mu^1 + i W_\mu^2) (W^{\mu 1} - i W^{\mu 2}) (v+h)^2$$

$$- i g_w W_\mu^3 + i g' B_\mu \rightarrow \frac{1}{8} (i g_w W_\mu^3 - i g' B_\mu) (-i g_w W^{\mu 3} + i g' B^\mu) (v+h)^2$$

and the mass terms are then quadratic in the fields

$$(2) \rightarrow \frac{g_w^2 v^2}{8} (W_\mu^1 W^{\mu 1} + W_\mu^2 W^{\mu 2}) = \frac{M_W^2}{2} (W_\mu^1 W^{\mu 1} + W_\mu^2 W^{\mu 2})$$

$$\text{with } M_W = \frac{1}{2} g_w v.$$

We find that the kinetic Lagrangian term can be expressed as

the matrix (in vacuum expectation value) in terms of

$$W^{\mu 3}, B^\mu \quad (W^{\mu 1}, W^{\mu 2} \text{ is diagonalized above})$$

$$|\partial_\mu \psi|^2 = \frac{v^2}{8} (W_\mu^3 \ B_\mu) \begin{pmatrix} g_w^2 & -g_w g' \\ -g_w g' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\rightarrow \left. \begin{aligned} Z_\mu &= \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu \\ A_\mu &= \sin \theta_w W_\mu^3 + \cos \theta_w B_\mu \end{aligned} \right\} \leftrightarrow \left\{ \begin{aligned} B_\mu &= \cos \theta_w A_\mu - \sin \theta_w Z_\mu \\ W_\mu^3 &= \sin \theta_w A_\mu + \cos \theta_w Z_\mu \end{aligned} \right.$$

such that $M_A = 0$; $M_Z = \frac{v}{2} \sqrt{g_w^2 + g'^2}$ as eigenvalues.

We can compare the M_W, M_Z masses to find

$$M_Z = \frac{M_W}{\cos \theta_w} \quad \text{such that} \quad \tan \theta_w = \frac{g'}{g_w}.$$

- One remaining issue: fermion masses...

We found before that this breaks Gauge symmetry due to the coupling of an $SU_L(2)$ doublet to a $SU_L(2)$ singlet.

$$\mathcal{L}_\psi = -g_e \left[(\bar{\psi}_L \ \bar{e}_L) \begin{pmatrix} \psi^+ \\ \psi^0 \end{pmatrix} e_R + \text{h.c.} \right]$$

* Couple the left chiral fields to the Higgs fields,
we can generate an overall singlet.

By acquiring a vacuum expectation value, this becomes the electron mass

$$g_e v \sim m_e$$

